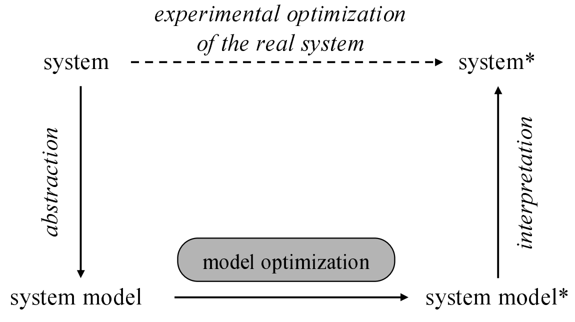
Introduction into Parameter Optimization of Simulation Models

Outline

- Motivation and objectives
- Specification of the model function
- The procedure of model optimization
- Real parameter and combinatorial optimization
- Constraints and basic problems of model optimization
- Properties of the model function
- Requirements on the applied optimization strategies
- Classification of optimization methods
- Essential aspects of local and global optimization

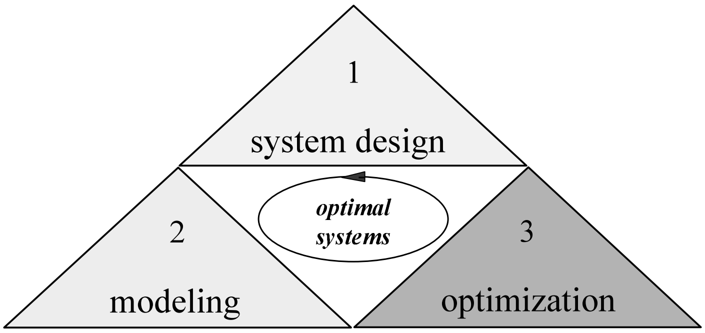
Motivation and Objectives

- Primary goal of system design and system tuning: **“optimal" systems**
- Alternatives for system optimization:

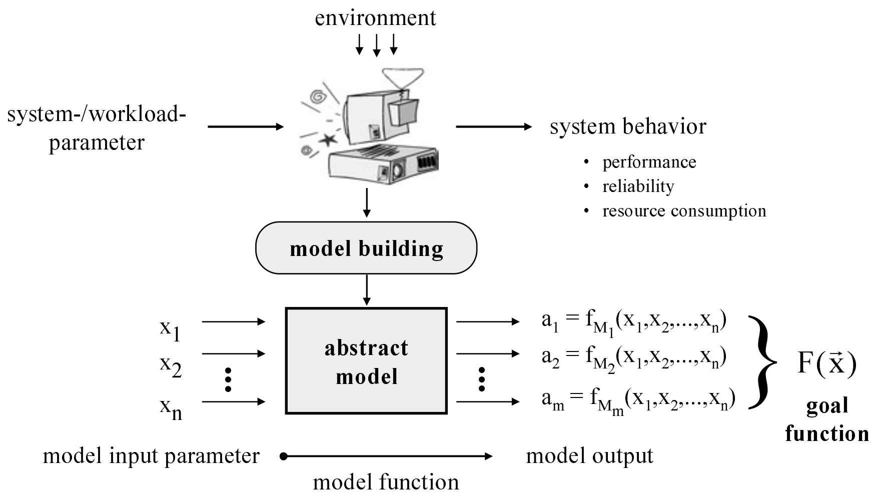


- **<u>Required:</u>** Qualified methods for global optimization of simulation models

Involved Domains

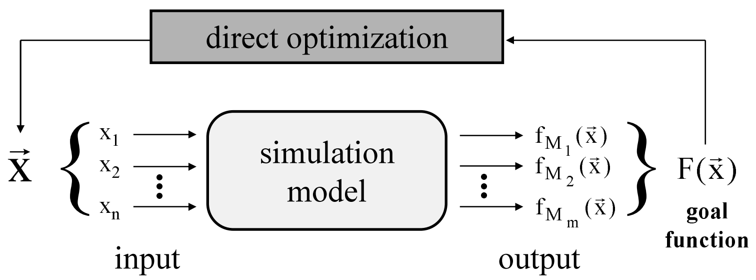


Specification of the Model Function



$f_M: D \subset R^n \to R^m$

The Procedure of Model Optimization



Required:

parameter vector $\vec{x}^* \in L$ with :

$$\forall \vec{x} \in L : F(\vec{x}) \circ F(\vec{x}^*), \quad \circ \in \{\leq, \geq\}$$

Real Parameter Optimization

Let F be the goal function and L be the search space:

$$F : L \subseteq R^n \rightarrow R, \quad L \neq \emptyset$$

$$L = \left\{ \vec{x} \in R^n \left| \begin{array}{l} h_i(\vec{x}) = 0, i \in \{1,..., p\} \\ g_j(\vec{x}) \leq 0, j \in \{1,..., q\} \end{array} \right. \right\}; \quad \text{with } p, q \in N$$

$h_i : R^n \rightarrow R$
$g_j : R^n \rightarrow R$ parameter restrictions

Required: Parameter vector $\vec{x}^* \in L$ with:

$$\forall \vec{x} \in L : F(\vec{x}) \circ F(\vec{x}^*), \quad \circ \in \{\leq, \geq\}$$

F^* : global optimum

$\vec{x}^*$: global optimum point

NP-hard

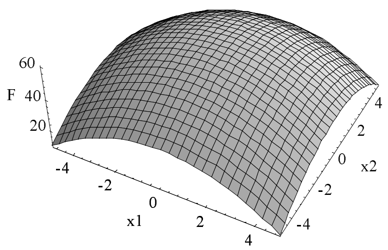
Constraints and Basic Problems of Model Optimization

- no additional analytical information available (black-box situation)
- expensive goal function evaluation through simulation
- stochastic inaccuracies
- high-dimensional search space with complex parameter restrictions
- multimodal goal function with many local and/or global optimum points

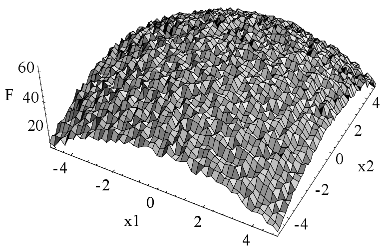
Vorlesung „Simulationstechnik"
(Dr. M. Syrjakow) 7

Properties of the Model Function

- analytical model
  - additional analytical information
  - high accuracy



- simulation model
  - only goal function values
  - stochastic inaccuracies



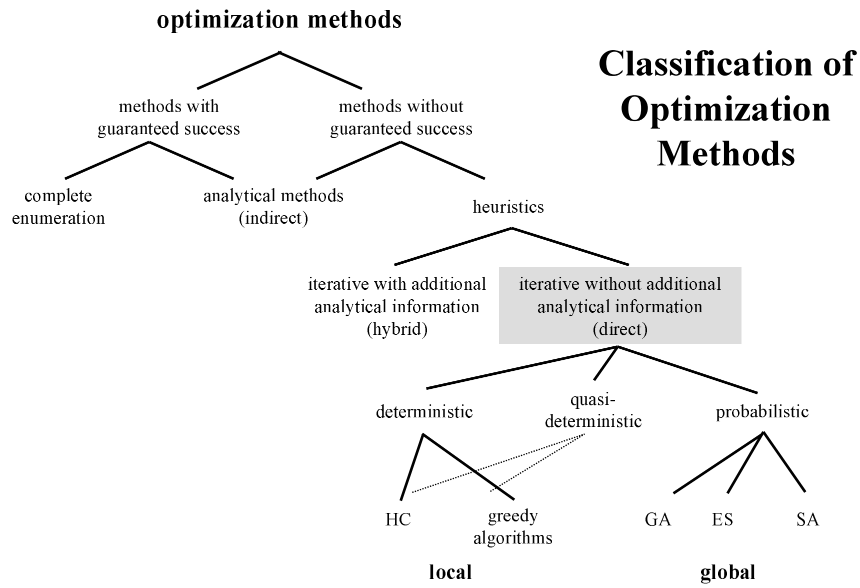
Vorlesung „Simulationstechnik"
(Dr. M. Syrjakow) 8

Parameter Optimization of Simulation Models

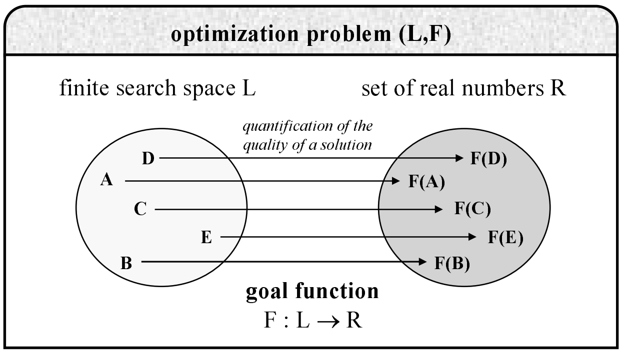
Requirements on the applied optimization strategies

- **no usage of additional analytical information**
- **qualification for global search**
  - high convergence speed
  - high approximation accuracy
  - high efficiency
  - robustness against stochastic inaccuracies
  - suitability for many kinds of goal functions
  - suitability for high numbers of model input parameters
    - easy handling
    - low implementation effort

Classification of Optimization Methods



Combinatorial Optimization



Required: problem solution $i^* \in L$

$\forall i \in L : F(i) \circ F(i^*) = F^*, \quad \circ \in \{\leq, \geq\}$

F^* : global optimum

i^* : global-optimally solution

Direct Local Optimization

- **Let**
  - **(L,F)** be a combinatorial optimization problem
  - $\eta : L \rightarrow 2^L$ be a neighborhood structure
  - i_{start} be a starting solution
- **Required**
  - problem solution $i^\wedge \in L$: $\forall j \in L_i$: $F(j) \circ F(i^\wedge)$, $\circ \in \{\leq, \geq\}$
- **Main problems**
  - definition of the neighborhood structure
  - choice of the starting solution

- **Basic algorithm**

```
procedure local_minimization;
begin
    choose_a_starting_solution(i_start ∈ L);
    i:=i_start;
    repeat
        generate_a_neighbouring_solution(j ∈ L_i);
        if F(j) < F(i) then i:=j;
    until F(j) ≥ F(i), ∀j ∈ L_i;
end;
```

Direct Global Optimization

Let **(L,F)** be a combinatorial optimization problem

Required

problem solution $i^{*} \in L$

$\forall j \in L : F(j) \circ F(i^{*}), \quad \circ \in \{\leq, \geq\}$

Optimization methods

- **complete enumeration**
- **heuristics, based on stochastic operators**

Main problems

- usually no a priori knowledge about the goal function available
- no basic algorithm
- no efficient criterion for the proof of global optimum points

Direct Optimization Methods

- **Local**
  - Hill-Climbing Strategies (HC)
    - Strategy of Hooke and Jeeves (Pattern Search PS)
- **Global**
  - Population based
    - Monte Carlo (MC)
    - Genetic Algorithms (GA)
    - Evolution Strategies (ES)
  - Point-to-Point
    - Simulated Annealing (SA)

Comparison of Direct Global and Local Optimization Methods

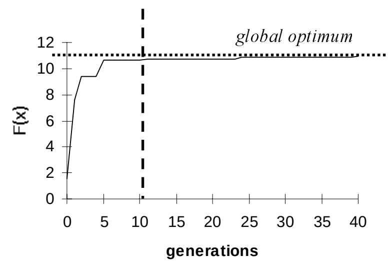
	local optimization methods	**global** optimization methods
Advantages	+ exact or at least ε -accurate localization of optimal solutions + high convergence speed + high efficiency	+ ability to escape from sub-optimal regions of the search space
Dis-advantages	- no escape from sub-optimal regions of the search space (the optimization result is determined by the starting solution)	- very low convergence speed especially in the neighborhood of optimal solutions - high optimization effort - uncertain quality of the optimization results

➔ for global optimization of simulation models not sufficiently qualified

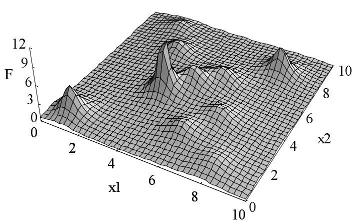
Example: Global Optimization

Applied optimization method:

Genetic Algorithms



Optimization problem



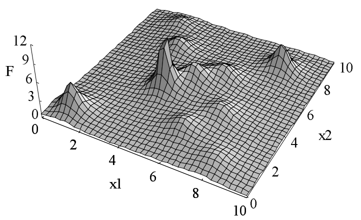
Shekel-Function

- **Advantage**
  + exploration of the search space
- **Disadvantage**
  - low convergence speed in the neighborhood of optimal solutions

Example: Local Optimization

Applied optimization method: Pattern Search

optimization problem



Shekel-Function

experi-ment	starting point	optimum point	goal function evaluations
1	(6,8)	F(8,8)≈5,37	42
2	(3,8)	F(2,9)≈2,05	44
3	(1,0)	F(1,1)≈5,23	35
4	(3,3)	F(4,4)≈11,0	35
5	(3,6)	F(3,7)≈3,13	37
6	(10,2)	F(8,1)≈1,92	47
7	(6,7)	F(6,6)≈3,58	47

Advantage

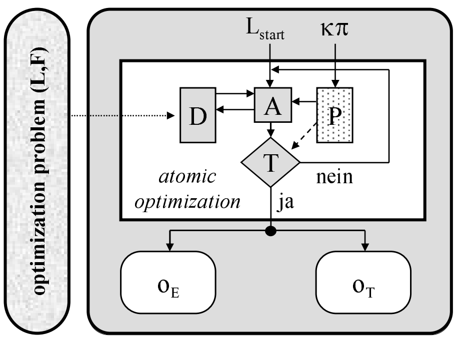
+ high convergence speed
+ ε-accurate localization (here ε = 0,01)

Disadvantage

- optimization success depends on the starting point

Components of an Atomic Direct Optimization Method

Basic structure



A	optimization algorithm
D	data structure
T	termination condition
P	control parameter

L_{start}	set of starting solutions
κπ	control parameter setting
o_E	optimization result
o_T	optimization trajectory