Hybrid Optimization Strategies

Outline

- Introduction
- Combined 2-phase optimization
- Multiple-stage optimization
- Efficient pre-optimization through goal function approximation
- REMO (REsearch Model Optimization Package)
- Evaluation of direct optimization methods
- Case-studies

Global Optimization of Simulation Models

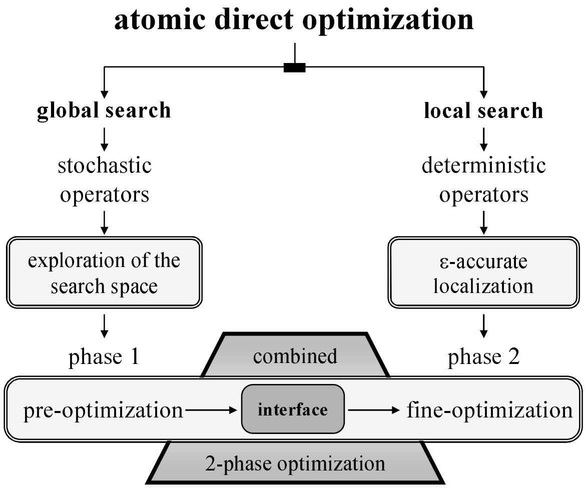
Main Difficulties

- black-box situation
- expensive model evaluation process
- stochastic inaccuracies
- high dimensional search space with complex parameter restrictions
- multimodal goal function with many local and/or global optimum points

Possible Objectives

- Improvement of an already known solution
  - local optimization
- Search for at least one globally-optimal solution
  - global optimization
- Systematic search for the most prominent extreme points
  - multiple-stage optimization

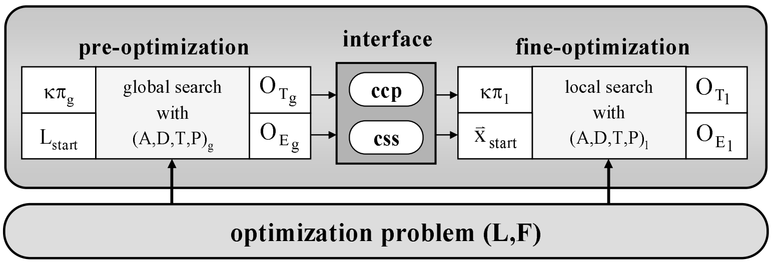
Basic Idea of Combined 2-Phase Optimization



Splitting of the Optimization Process into two Phases

- **pre-optimization**
  - rough exploration of the search space for promising regions
  - abstraction of unessential details of the goal function surface
- **fine-optimization**
  - closer consideration of a certain search space region
  - ε-accurate localization of the optimum point in this region

Combined 2-Phase Optimization

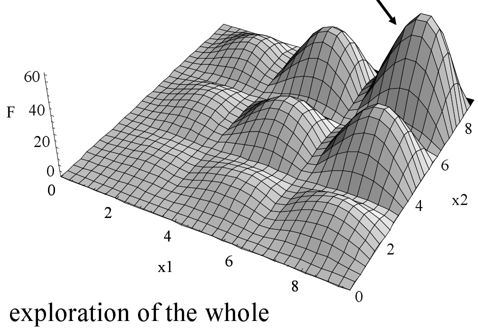


Main problems

- parameterization of the global optimization strategy ($\kappa\pi_g$)
- switching from pre- to fine-optimization (T_{PO})
- computation of control parameter settings from optimization trajectories (ccp)
- choice of starting solutions (css)

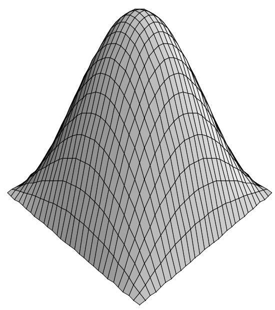
Example

pre-optimization with Genetic Algorithms



exploration of the whole search space

fine-optimization with Pattern Search



local search in a specific search space region

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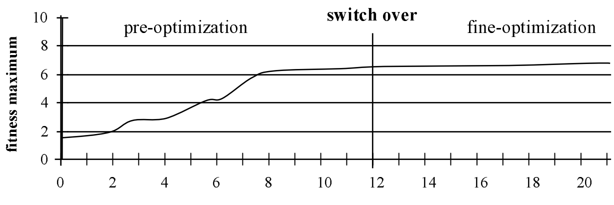
Switching from Pre- to Fine-Optimization

Problem: The switching has to occur "in time"

Solution: Definition of heuristical switching criterions based on

- the number of already generated search points
- search point constellations
- the development of the best goal function value found so far
- a priori knowledge about the optimization problem

Example:



➢ logical combination of several switching criterions

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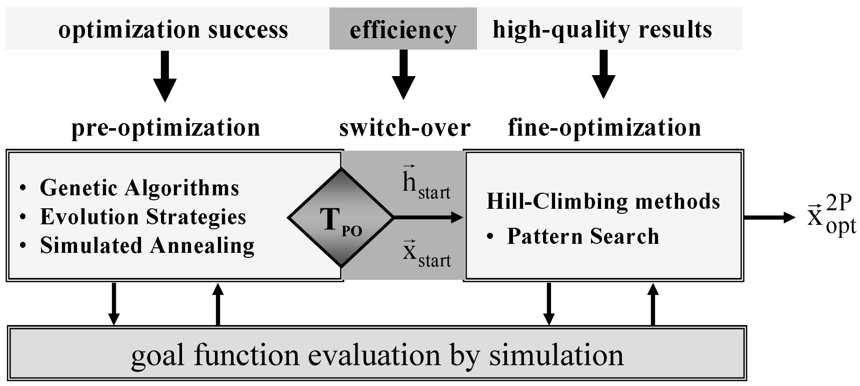
Parameterization of Fine-Optimization

- choice of starting solutions
  - one point start (choice of exactly one starting point)
  - multiple-point start (choice of more than one starting point)
- computation of control parameter settings (initial step sizes)
  - cluster analysis of the pre-optimization trajectory

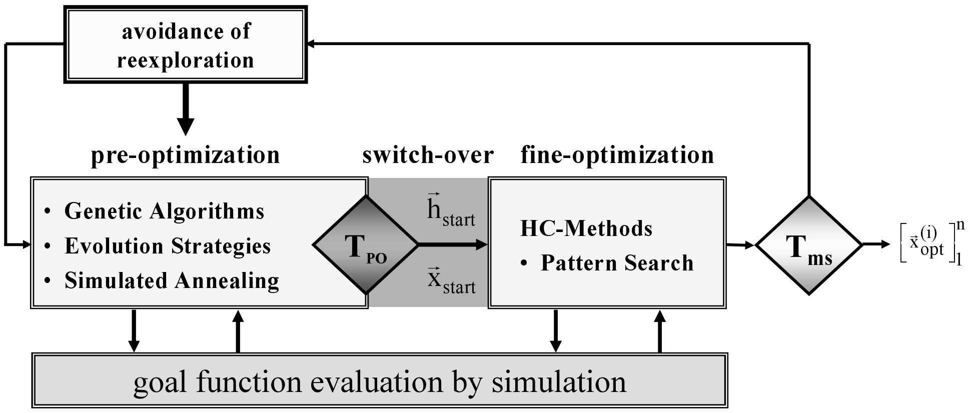
Combined 2-Phase Optimization

for global parameter optimization of simulation models



Multiple-Stage Optimization

for the systematic search for the most prominent extreme points



Avoidance of Reexploration

- Problem: Repeated localization of already found optimum points
- Required: Method which makes already explored regions of the search space unattractive for the pre-optimization strategy

➢ Solution: Attractiveness values $av \in [0,1]$

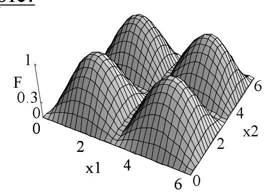
$$av(\vec{x}) = \prod_{i=1}^{k}\left[1 - \frac{1}{(1+\alpha \cdot d_i)^{\beta}}\right], \quad d_i = \sqrt{(\vec{x} - \vec{p}_i)^2}$$

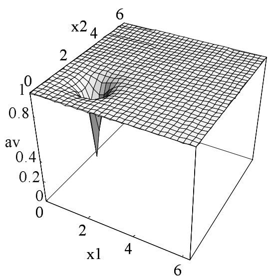
$\alpha, \beta \in R$ (scaling factors),

p_i : found optimum points,

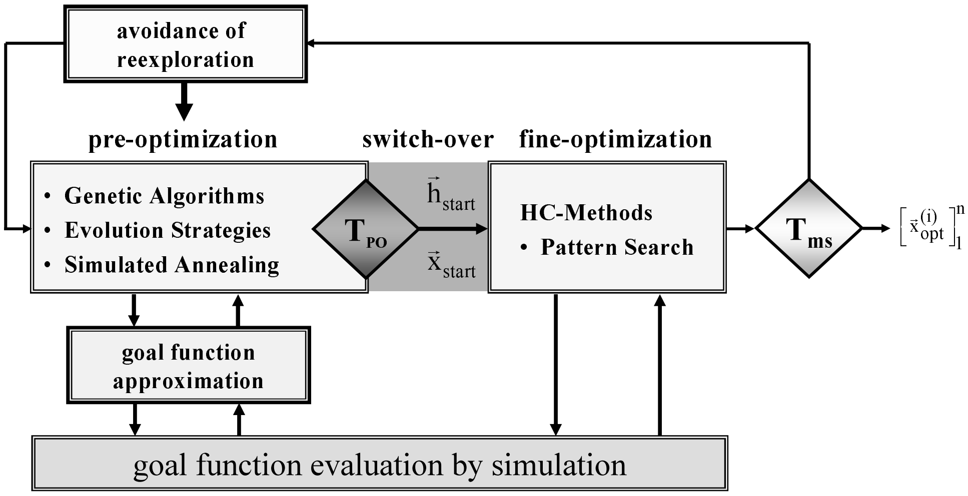
k : number of already found optimum points.

Example:





Efficient Pre-Optimization through Goal Function Approximation



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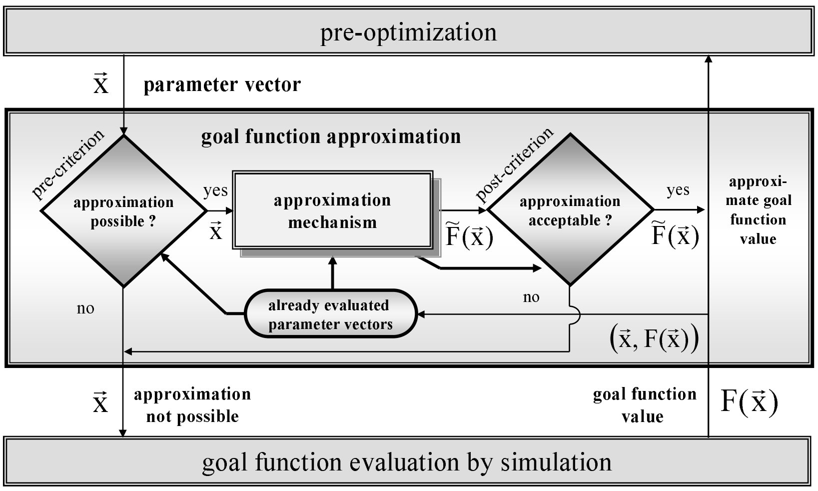
Goal Function Approximation



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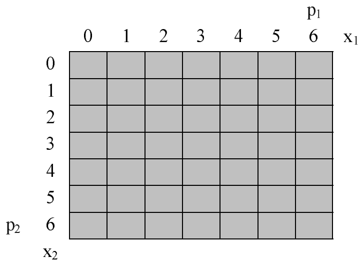
Grid-Based Approximation

Subdivide the search space into sectors of equal size



$$S = \{(s_1, s_2, ..., s_n) \in N^n \mid s_i \in \{0, ..., p_i\},$$
$$p_i \in N;\; i \in \{1, ..., n\};\, n \in N\}$$

Keep a statistic for each sector s during the pre-optimization process:

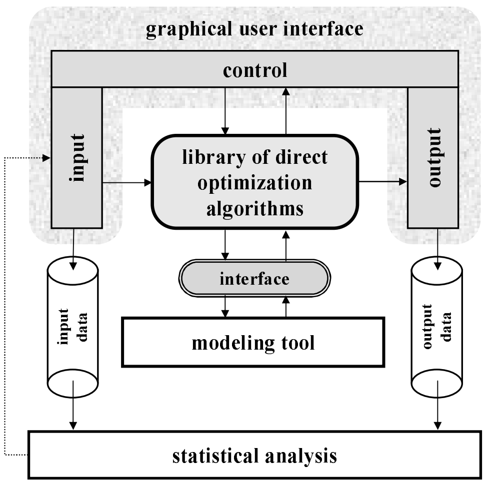
- e^s: number of entries
- F^s_{avg}: average goal function value
- F^s_{max}: maximum goal function value
- F^s_{min}: minimum goal function value

Use the following approximation criterion for a search point from e^s:

If $e^s \leq e^s_{max}$ then evaluate the simulation model else approximate

REMO (REsearch Model Optimization Package)

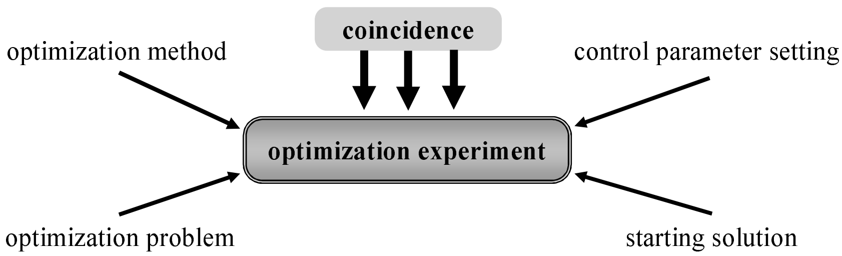
Architecture



Evaluation of Direct Optimization Methods

- influences on the performance
  optimization problem, solution method, parameterization, coincidence
- performance measures
  optimization success, quality of the optimization results, optimization effort
- test problems
  simulation models, mathematical test problems
- optimization experiments
  planning, execution, evaluation of the results

Influences on the Performance of Direct Optimization Methods



Performance measures

- optimization success
- quality of the optimization result
- optimization effort

No Free Lunch Theorem

- All global optimization methods have the same performance when the performance is evaluated regarding all possible optimization problems.

  in other words:

- When method A is better than method B on some specific optimization problems there must exist some other optimization problems where method B has a better performance behaviour.

Consequences

- No global optimization method is clearly superior to all the other ones
- A priori knowledge about the optimization problem is of great importance

Wolpert, D.H.; Macready, W.G.: *No Free Lunch Theorems for Optimization*; in IEEE Transactions on Evolutionary Computation, Vol. 1, No. 1, April 1997, pp. 67-82.

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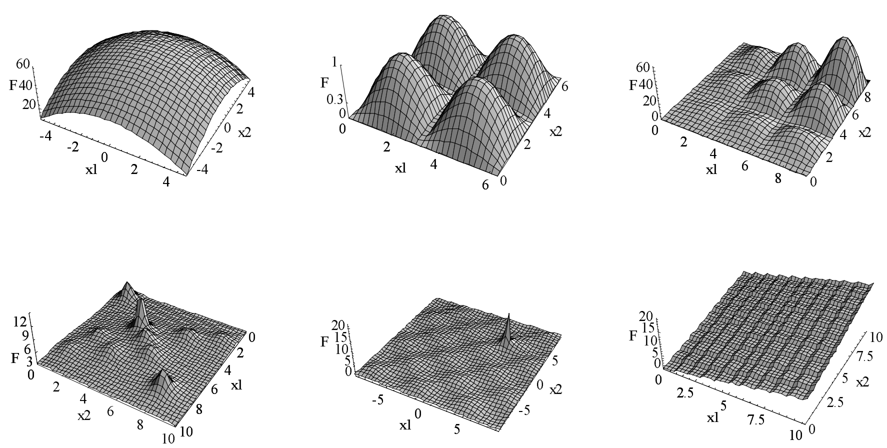
Mathematical Test Functions



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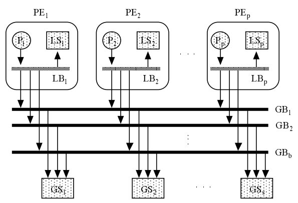
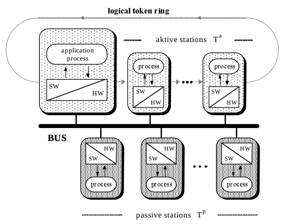
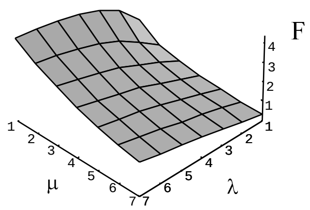
Simulation-Based Goal Functions

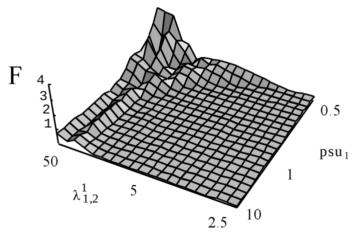
multiprocessor system



tokenbus system





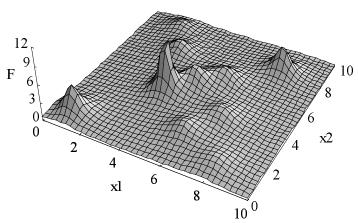


2-Dimensional Shekel-Function

test function: $F(\vec{x}) = \sum_{i=1}^{10} 1/f_i$

$$f_1(\vec{x}) = (x_1-4)^2 + (x_2-4)^2 + 0{,}1$$
$$f_2(\vec{x}) = (x_1-1)^2 + (x_2-1)^2 + 0{,}2$$
$$f_3(\vec{x}) = (x_1-8)^2 + (x_2-8)^2 + 0{,}2$$
$$f_4(\vec{x}) = (x_1-5)^2 + (x_2-5)^2 + 0{,}3$$
$$f_5(\vec{x}) = (x_1-6)^2 + (x_2-6)^2 + 0{,}4$$
$$f_6(\vec{x}) = (x_1-3)^2 + (x_2-7)^2 + 0{,}4$$
$$f_7(\vec{x}) = (x_1-6)^2 + (x_2-2)^2 + 0{,}5$$
$$f_8(\vec{x}) = (x_1-7)^2 + (x_2-3{,}6)^2 + 0{,}5$$
$$f_9(\vec{x}) = (x_1-2)^2 + (x_2-9)^2 + 0{,}6$$
$$f_{10}(\vec{x}) = (x_1-8)^2 + (x_2-1)^2 + 0{,}7$$

function surface



constraints: $0 \le x_j \le 10,\ j \in \{1,2\}$

global optimum points: 1

$\vec{x}^* = (4,4);\ F^* \approx 11{,}0309$

local optimum points: 9

Search Space

GA parameter encoding: binary vector (10 Bit for each parameter)

dimension	number of possible solutions
1	$2^{10} = 1024$
2	$2^{20} = 1{,}048576 \cdot 10^{6}$
3	$2^{30} = 1{,}073741^{9}$
4	$2^{40} = 1{,}099511 \cdot 10^{12}$
5	$2^{50} = 1{,}125899 \cdot 10^{15}$
6	$2^{60} = 1{,}152921 \cdot 10^{18}$
7	$2^{70} = 1{,}180591 \cdot 10^{21}$
8	$2^{80} = 1{,}208925 \cdot 10^{24}$
9	$2^{90} = 1{,}237940 \cdot 10^{27}$
10	$2^{100} = 1{,}267650 \cdot 10^{30}$

= 1.267.650.000.000.000.000.000.000.000.000

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Optimization Results

GA+HC in multiple-stage mode

2-dimensional Shekel-function

st.	optimum point	GA	HC
1	F(1,1) ≈ 5.2372	65/5	42
2	F(4,4) ≈ 11.0309	83/5	66
4	F(6,2) ≈ 2.8176	74/6	47
5	F(7,3.6) ≈ 2.9045	56/4	46
6	F(6,6) ≈ 3.5852	61/4	57
7	F(8,8) ≈ 5.3725	42/4	49
8	F(5,5) ≈ 4.7598	53/4	49
9	F(3,7) ≈ 3.1384	61/5	50
19	F(2,9) ≈ 2.0551	46/5	47
20	F(8,1) ≈ 1.9214	41/3	45
	∑ =	**1080**	**1021**

4-dimensional Shekel-function

st.	optimum point	GA	HC
1	F(4,4,4,4) ≈ 10.5357	117/5	143
2	F(6,6,6,6) ≈ 2.8710	187/10	141
3	F(6,2,6,2) ≈ 2.4216	170/8	167
4	F(1,1,1,1) ≈ 5.1282	137/7	149
7	F(3,7,3,7) ≈ 2.8066	149/7	126
8	F(8,8,8,8) ≈ 5.1754	94/4	148
11	F(2,9,2,9) ≈ 1.8594	105/4	153
12	F(7,3.6,7,3.6) ≈ 2.4272	177/9	129
14	F(5,5,3,3) ≈ 3.8353	113/5	136
16	F(8,1,8,1) ≈ 1.6765	251/12	117
	∑ =	**2343**	**2294**

demanded accuracy: 0,01

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Optimization Results

GA+HC with multiple-point start in multiple-stage mode

2-dimensional Shekel-function

st.	optimum point	GA	HC
1	F(7,3.6) ≈ 2.9045	64/5	54
2	F(4,4) ≈ 11.0309	-	65
3	F(4,4) ≈ 11.0309	-	41
4	F(8,8) ≈ 5.3725	58/4	49
5	F(8,8) ≈ 5.3725	-	47
6	F(1,1) ≈ 5.2372	87/7	45
7	F(6,6) ≈ 3,5852	-	40
8	F(6,2) ≈ 2.8176	63/4	47
9	F(4,4) ≈ 11.0309	-	62
10	F(5,5) ≈ 4.7598	-	54
11	F(5,5) ≈ 4.7598	65/4	56
12	F(4,4) ≈ 11.0309	-	61
13	F(6,6) ≈ 3.5852	-	45
14	F(8,1) ≈ 1.9214	-	54
15	F(3,7) ≈ 3.1384	-	48
16	F(2,9) ≈ 2.0551	-	52
	∑ =	**337/24**	**820**

optimization stage

goal function evaluations/generation | goal function evaluations

demanded accuracy: 0,01

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Optimization Results

GA+HC with multiple-point start in multiple-stage mode

4-dimensional Shekel-function

st.	optimum point	GA	HC
1	F(8,1,8,1) ≈ 1.6765	101/4	109
2	F(4,4,4,4) ≈ 10.5357	132/5	154
3	F(7,3.6,7,3.6) ≈ 2.4272	-	140
4	F(7,3.6,7,3.6) ≈ 2.4272	-	197
5	F(4,4,4,4) ≈ 10.5357	-	184
6	F(1,1,1,1) ≈ 5.1282	154/6	134
7	F(7,3.6,7,3.6) ≈ 2.4272	178/7	123
8	F(4,4,4,4) ≈ 10.5357	-	151
9	F(6,2,6,2) ≈ 2.4216	-	132
10	F(5,5,3,3) ≈ 3.8353	-	137
11	F(1,1,1,1) ≈ 5.1282	157/6	137
12	F(6,6,6,6) ≈ 2.8710	291/12	119
13	F(7,3.6,7,3.6) ≈ 2.4272	128/5	139
14	F(7,3.6,7,3.6) ≈ 2.4272	-	146
15	F(6,2,6,2) ≈ 2.4216	-	121
16	F(8,8,8,8) ≈ 5.1754	204/8	139
17	F(3,7,3,7) ≈ 2.8066	-	131
18	F(3,7,3,7) ≈ 2.8066	-	138
19	F(2,9,2,9) ≈ 1.8594	-	113
	∑ =	**1345/53**	**2644**

demanded accuracy: 0,01

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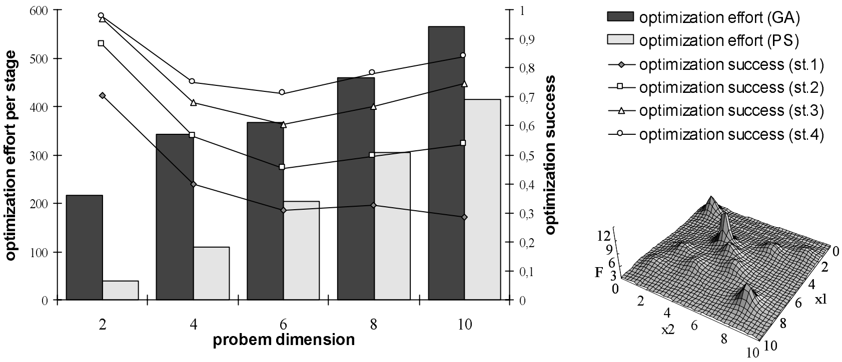
Results

optimization method: combined 2-phase strategy (multiple-stage mode)

- pre-optimization: Genetic Algorithms (GA)
- fine-optimization: Pattern Search (PS)



optimization experiments: 200 • optimization stages: 20 • demanded accuracy: 0,01

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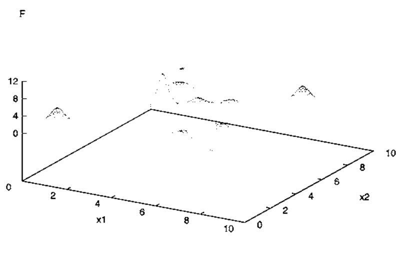
Genetic Algorithms

pre-optimization results



200 x 20 experiments
with avoidance of reexploration

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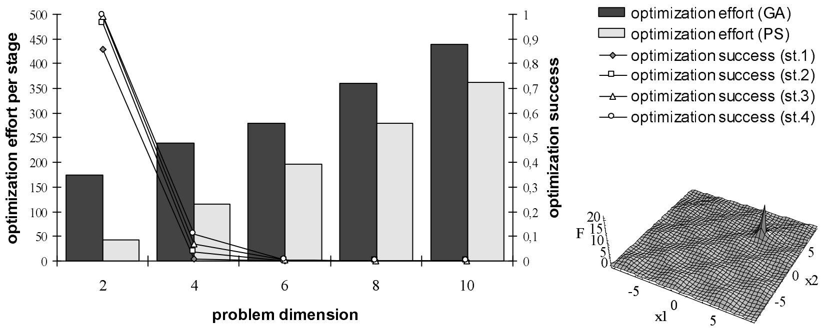
Results

optimization method: combined 2-phase strategy (multiple-stage mode)

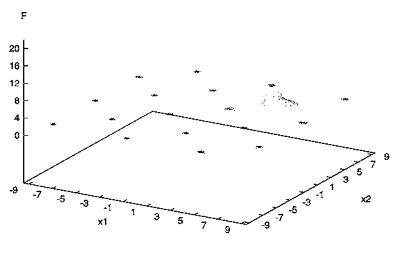
- pre-optimization: Genetic Algorithms (GA)
- fine-optimization: Pattern Search (PS)



optimization experiments: 200 • optimization stages: 20 • demanded accuracy: 0,01

Genetic Algorithms

pre-optimization results



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Results

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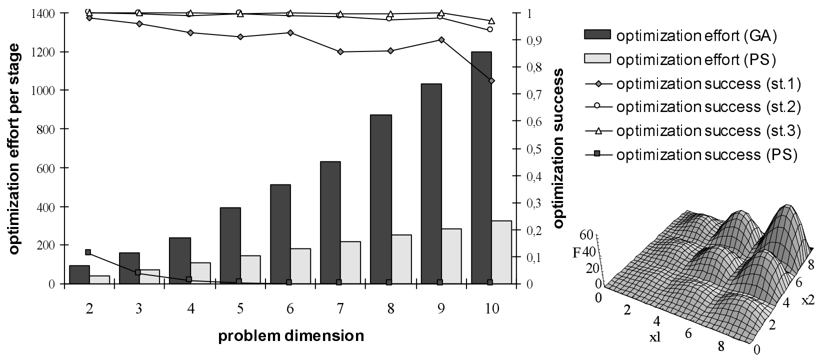

optimization experiments: 200 • optimization stages: 20 • demanded accuracy: 0,01

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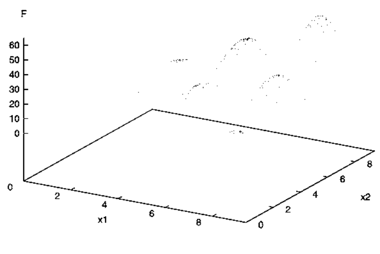
Genetic Algorithms

pre-optimization results



200 x 20 experiments
with avoidance of reexploration

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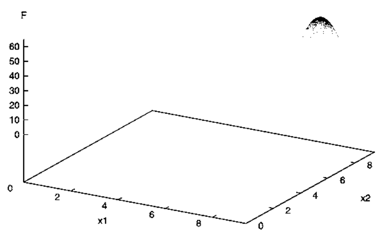
Genetic Algorithms

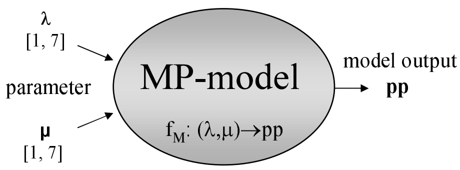
pre-optimization results



200 x 20 experiments
without avoidance of reexploration

Case-Study (Model Optimization)

- **optimized system:** multiprocessor system
- **question:** which workload (λ,μ) causes an optimal system performance (processing power pp)?

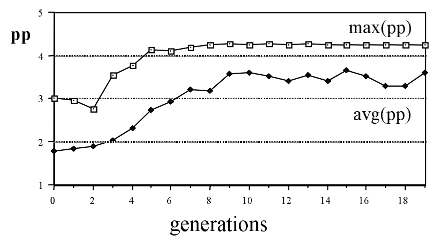


➢ **Solution**

GA parameter

- population size: 20
- binary vector length: 20 (10/10)
- crossover probability: 0,5
- mutation probability: 0,02

GA optimization



Results

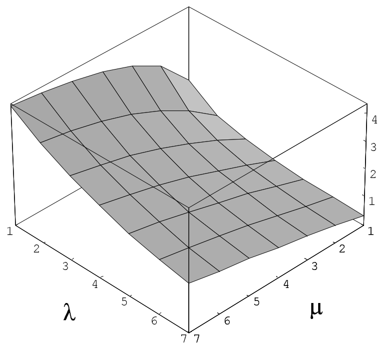
- parameter encoding

20 Bit

1001011010	1011011010
λ	μ

- possible solutions
  2^{20} = **1.048.576**
- model evaluations (8 generations)
  8 · 20 = **160**

model function f_M



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Additional Information

- **Literature**
  - Syrjakow, M.; Szczerbicka, H.: Efficient Parameter Optimization based on Combination of Direct Global and Local Search Methods; in Evolutionary Algorithms; L. Davis, K. De Jong, M. Vose, L.D. Whiteley (eds.); Springer Verlag, New York; will be published in 1999.
  - Syrjakow, M.; Szczerbicka, H.: Efficient Methods for Parameter Optimization of Simulation Models; Proceedings of the 1st World Congress on Systems Simulation (WCSS'97), Singapore, Republic of Singapore, September 1-3, 1997.
- **Information on the Web**
  - http://goethe.ira.uka.de/people/syrjakow/

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